Modelling performance of the Classical Weibull, Classical Exponential and Some Generalized Weibull and Exponential Distributions based on failure data sets.

Realman Evans Obele[1]
&
Barinaadaa John Nwikpe[2]
Department of Mathematics/Statistics[1&2],
Ignatius Ajuru University of Education,
Port Harcourt, Nigeria.

Abstract
This study was carried out to determine the performance of the classical Weibull and exponential distributions compared with the performance of some compound distributions, such as the Generalized Weibull, Three-Parameter Weibull, Weibull inverse Exponential distribution, Exponential Generalized and Inverse Exponential distribution using fourfailure data sets. R programming codes were developed for the maximum likelihood estimate of the parameters of the models. The goodness of fit tests is used to compare fitted model and determine how well the distributions fit a given data set. To achieve this, the following goodness of fit criteria were considered; -2 Likelihood, Akaike Information Criterion (AIC), Akaike Information Criterion Corrected (AICC) and Bayesian Information Criterion (BIC). The findings of the study revealed that the generalized Weibull distribution performed better than the classical Weibull and Classical exponential distribution.

Keywords: Weibull Distribution, Exponential Distribution, Parameter Estimation, Generalization and Goodness of fit.

Introduction

Efficient maintenance practices are strategic for businesses whose success is reliant on high-performance equipment. Asset-intensive sectors confront difficulties in appropriate prediction of the active life circle of its equipment. One aspect of maintenance and replacement management is interpreting data for the purpose of managing the equipment as effectively as possible. To achieve this, the gathering of a large amount of data is always needed for the purpose of analysis; useful data comes from a variety of sources, including customers, maintenance and operating personnel. All of these data is processed in a systematic way with the purpose of making sound decisions (Levitin, 2000, Tsai et al., 2001). A manufacturer's maintenance costs account for approximately 15% to 40% of the cost of the product manufactured, however one out of three of all maintenance costs are attributed to unnecessary or wrong maintenance procedures (Murthy et al.,2004). Experts in all businesses are confronted with this challenge of making proper maintenance and replacement decision; regretfully, this problem is often not solved efficiently. This growing desire for efficiency has necessitated the employment of Mathematical models, particularly for large scale equipment and components to prevent long-term shutdowns of critical equipment such as haul truck engines, mill liners, and shovel swing transmissions.

In the past, cost optimization was a popular principle for planning replacements or maintenance of equipment component. Conversely, this technique did not take into account factors such as the reliability of the system and the probability of a sudden breakdown, which could have a significant impact on preventive maintenance decisions (Wang &Coit, 2005). The concept of reliability and likelihood suggest that modelling of replacement and *maintenance could be enhanced by introducing the notion of probability. Adopting probability models for maintenance is one of the answers to these problems since equipment maintenance has become more important in order to assure availability, functional performance, and reliability of equipment.*

There exist a lot of literature on system and component maintenance, replacement models as well as the statistical reliability models all geared towards improving system or component performance (Yadav et al., 2003); (Rosss, 1980); (Bilinton, et al., 2000); (Connor, 1991); (Jarrdine, 2003). The majority of maintenance and renewal decisions, on the other hand, are based on past experience and expert estimates. As earlier mentioned, expert judgment-based evaluation approaches have been used in the past, the technique is based on the experts' experience with system/component failure patterns.

To achieve a balance of operational and economical goals, decision-making models for dealing with important replacement problems should incorporate reliability methodology, and different probabilistic or mathematical models. On the other hand, quantitative assessment can be utilized for two primary reasons: past performance evaluation and future performance prediction. Historical evaluation, also known as past performance assessment, examines the system's actual performance over a specific time period using suitable sets of reliability parameters, the system's failure history is also examined (Alaswad, 2012; Anders et al., 2001; Cho &Parlar, 1991).

To be able to appraise the previous performance of the system, historic evaluation necessitates the gathering and analysis of historical relevant data such as, failure incidences, durations of breakdowns and reasons for breakdowns. In quantitative terms, the reliability model parameters is being evaluated and quantified numerically using some mathematical models.

A predictive assessment, on the other hand, can be used to analyze the system's future performance. After gathering the necessary lifetime data to be used for the study, it becomes pertinent to select an appropriate mathematical model for estimation and analysis to be reasonable. Consequently, a variety of mathematical and statistical models have been developed, including the widely used Weibull distribution, Exponential distribution, Gaussian distribution, Gamma distribution, Lognormal distribution, and cost optimization models amongst others. The modification of some standard models have also been done by some researchers.

The classical Weibull and exponential distributions are among the mathematical models popularly used for modelling failure data. According to Tang (2004) the failure time of most systems were believed to follow an exponential distribution before the 1980s since it has less complex mathematical form with tractable statistical characteristics. Yong (2004) discovered that major short fall of the Weibull probabilistic models is that the failure rate of a system after multiple repair does not follow the Weibull model. Thus, an optimal replacement policy might not be obtained if the Weibull classical distribution is used to model such system. Nwikpe and Didi (2021) also noted that the constant failure rate of the exponential distribution makes it inappropriate for modelling most system, since the failure rate might be a function of time. One of its important properties is the so-called no-memory property which applies to products with an exponential lifetime distribution; the history of the past behaviour of the system is inconsequential. The second feature of the exponential distribution is that its failure rate is constant, which is one its shortfalls.

To model such failure time data, the Weibull distribution is often employed. In the last few years, various studies have been carried out on the generalization/modification of the classical Weibull distribution. The Weibull distribution has also been studied extensively and applied in a variety of fields. Nevertheless, the assertion of a monotonic rate of failure may not always be realistic real-life setting. The present study is carried outto assess the performance of some modified Weibull an exponential distribution for failure data sets.

Methodology

Failure Density

The failure density or in amore broad perspective the probability density function (pdf) of a failure data is described mathematically as;

$$f(x.) = \lim_{\partial \to 0} \frac{t. - \frac{\partial}{2} < T < t. + \frac{\partial}{2}}{\partial} \quad (1)$$

In equation (1) the function *f(x)* approximates the probability of failure in the time frame

$$t. - \frac{\partial}{2}, t. + \frac{\partial}{2}.$$

The probability of reaching an age between $t_{i.}$ *and* $t_{j,}$ $t_{i.} < t_{j.}$ *is*

$$\Pr(t_i \le T \le t_j) = \int_{t_i}^{t_j} f(t)\,dt \quad (2)$$

The Lifetime Distribution Function

Another model which might be utilized in the description of the failure of a system is the cumulative lifetime distribution also called the failure function or lifetime function defined as

$$F(t) = P(T \le t) (3)$$

Precisely for a totally new system or just produced unit for example, given that the unit is beginning at age $t > 0$ is given by

$$\Pr(T \le t_j) = \int_{o}^{t} f(x)dx \quad (4)$$

The lifetime distribution function is given by equation (4), which gives the probability of having a life span of at most length t.

Any function $F(T)$ may be a cumulative distribution function of a lifetime variable if it satisfies the following properties:

$$\lim_{t \to 0} F(t), = 0$$

$$\lim_{t \to \infty} F(t) = 1$$

$$F(t_i) \ge F(t_j), \forall\ s\ t_i \ge t_j \quad (5)$$

$$0 \le F(t) \le 1.$$

Some Selected Probability Models and their Reliability Function

In this study, we examined the performance of some generalized Weibull and exponential and the TPAN distribution for some selected failure data sets.

The Weibull Distribution

The following is the probability density function of the classical two parameter Weibull distribution:

$$f(x.) = \left(\frac{\beta}{\theta}\right)\left(\frac{x.}{\theta}\right) \exp\left\{-\left(\frac{x.}{\theta}\right)^{\beta}\right\}$$

$$x. \ge 0, \theta > 0, \beta > 0 \quad (6)$$

With cumulative distribution function

$$F(x.) = 1 - \exp\left\{-\left(\frac{x.}{\theta}\right)^{\beta}\right\}$$

$$x. \ge 0, \theta > 0, \beta > 0 \quad (7)$$

Where x is the random variable representing the failure times or duration of failure of an equipment

θ and β are the model parameters

β = shape parameter

θ = scale parameter

This distribution, introduced by Weibull (1957) in Pedro et al., (2018), has useful mathematical qualities, and its failure occurs in a variety of settings (Manton &Yashin, 2006). McCool (2012) went into great detail about its application in reliability.

The Exponential Distribution

The exponential distribution's distribution function is given by

$$F(x) = 1 - e^{-\theta x} (8)$$

With probability density function

$$f(x) = \theta e^{-\theta x} \quad (9)$$

The Three- Parameter Weibull Distribution

Mathematically the density function of the three Weibull Distribution is defined as:

$$f(t.) = \beta\alpha^{-\beta}\left(\frac{t.-\theta,}{\alpha}\right)^{\beta-1} exp^{\left(-\left(\frac{t-\theta.}{\alpha}\right)^{\beta}\right)},$$

$$\alpha > 0, \beta > 0,\ \ \theta > 0 \qquad (10)$$

Where α is the location parameter?

$\alpha > 0, \beta > 0,$ and $\theta > 0$
are the parameters of the distribution.
With corresponding Cumulative Distribution Function (CDF)

$$F(t) = 1 - exp^{\left(-\left(\frac{t-\theta.}{\alpha}\right)^{\beta}\right)} \qquad (11)$$

The Weibull Inverse Exponential Distribution

A random variable t is said to follow the Weibull Inverse Exponential Distribution if its probability density function is given as;

$$f(x) = \alpha\beta\vartheta x^{-2}\frac{\left[\exp\left(-\frac{\vartheta}{x}\right)\right]^{\beta}}{1-\left[\exp\left(-\frac{\vartheta}{x}\right)\right]^{\beta+1}}$$

$$exp\left[-\alpha\left(\frac{\exp\left(-\frac{\vartheta}{x}\right)}{1-\exp\left(-\frac{\vartheta}{x}\right)}\right)^{\beta}\right] \qquad (12)$$

$$> 0, \vartheta > 0, \beta > 0$$

With cumulative distribution function given as;

$$F(y;\beta,\vartheta,\alpha) = 1 - exp\left[-\alpha\left(\frac{\exp\left(-\frac{\vartheta}{x}\right)}{1-\exp\left(-\frac{\vartheta}{x}\right)}\right)^{\beta}\right] \qquad (13)$$

Exponentiated Generalized Inverse Exponential Distribution (EGIED)

The failure density function of the exponentiated inverse exponential distribution is given by:

$$f(t) = \frac{\theta\alpha\beta}{t^2}\exp\left(-\frac{\theta}{t.}\right)xp\left(-\frac{\theta}{t}.\right)$$

$$\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha-1}\left(1-\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha}\right)^{\beta-1} \qquad (3.45)$$

$$t > 0, \beta > 0, \theta > 0$$

With CDF as:

$$F(t) = \left(1-\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha}\right)^{\beta-1} \qquad (14)$$

The reliability function is obtained as follows:

$$R(t) = 1 - \left\{1 - exp\left[-\beta\left(\frac{exp\left(-\frac{\theta}{t}\right)}{1-exp\left(-\frac{\theta}{t}\right)}\right)\right]\right\} \qquad (15)$$

The hazard rate function of (14) is given as follows:

$$H(t) = \frac{\theta\frac{\alpha\beta}{t^2}\exp\left(-\frac{\theta}{t.}\right)xp\left(-\frac{\theta}{t}.\right)\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha-1}\left(1-\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha}\right)^{\beta-1}}{\left\{1-exp\left[-\beta\left(\frac{exp\left(-\frac{\theta}{t}\right)}{1-exp\left(-\frac{\theta}{t}\right)}\right)\right]\right\}}$$

Method of Parameter Estimation

In order to obtain the unknown failure parameter parameters resulting from the failure density function, the Maximum-Likelihood Estimation (MLE) method will be adopted. Although, other approaches for example, Method of Moments (MOM), Least Squares Estimation and Bayesian Estimation also exist for parameter estimation. The choice of the MLE method amongst other methods is based on the following reasons:

i). It has been proven mathematically that the method of MLE has lower variance than the other methods in the class of unbiased estimators.

ii) As sample gets larger, maximum likelihood estimates are often minimum variance estimator unbiased.

iii) It offers a consistent solution to parameter difficulties. And it is, it can be designed to solve a wide range of estimation problems.

iv) The algorithms for calculating the maximum likelihood estimates for the vast majority of widely known distributions are easily accessible and installed in a variety of major Mathematical software.

As a result, the work is simplified and the computational complexity associated with the MLE is reduced.

The MLE is used to calculate the joint failure density of a component. If $f(y;\theta)$ is a failure density function, where y represents the failure time of a component, then the likelihood function is $L(y1, y2, y3,...yn;\theta)$ as a function of θ is given by (Hoel, 1954) in Palelum (2007) as;

$$L(y_1, y_2, y_3 \ldots, y_n; \theta) = f(y_1;\theta) \times f(y_2;\theta) \times \ldots\ldots \times f(y_n;\theta) \quad (16)$$

Equation (3.14) can also be written as:

$$L(y_1, y_2, y_3 \ldots, y_n; \theta) = \prod_{i=1}^{n} \left(f(y_i;\theta)\right) \quad (17)$$

Parameter Estimation

Maximum Likelihood Estimate of the Parameters of the Weibull Distribution

$$L(t;\theta) = \prod_{i=1}^{n} \left(f(x_i;\theta)\right)$$

$$L(x.;\beta,\theta) = \prod_{i=1}^{n} \left(\frac{\beta}{\theta}\right)\left(\frac{x.}{\theta}\right) \exp\left\{-\left(\frac{x.}{\theta}\right)^{\beta}\right\}$$

$$= \left(\frac{\beta}{\theta}\right)^{n} \prod_{i=1}^{n} \square \left(\frac{x.}{\theta}\right) \exp \sum_{i=1}^{n} \square \left\{-\left(\frac{x.}{\theta}\right)^{\beta}\right\}$$

$$\log L(x.;\beta,\theta) = n\log\beta - n\log\theta + \sum_{i=1}^{n} \square \left\{-\left(\frac{x.}{\theta}\right)^{\beta}\right\} = 0 \quad (18)$$

The solution of equation (11) gives the estimates of the model parameters. However, the equation cannot be solved analytically thus, was solved numerically using R programming with some data set.

Maximum Likelihood Estimate of the Weibull Inverse Exponential Distribution

$$L(x_i;\theta) = \prod_{i=1}^{n} \left(f(y_i;\theta)\right) \qquad i = 1,2,3,\ldots,n$$

$$= \prod_{i=1}^{n} \left(\alpha\beta\vartheta x_i^{-2} \frac{\left[\exp\left(-\frac{\vartheta}{x_i}\right)\right]^{\beta}}{1-\left[\exp\left(-\frac{\vartheta}{x_i}\right)\right]^{\beta+1}} \exp\left[-\alpha\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta}\right]\right)$$

$$= \alpha^n \beta^n \vartheta^n \prod_{i=1}^{n} \left(x_i^{-2}\right) \left(\frac{\left[\exp\left(-\frac{\vartheta}{x_i}\right)\right]^{\beta}}{1-\left[\exp\left(-\frac{\vartheta}{x_i}\right)\right]^{\beta+1}} \right)^{n} \left(exp\left[-\alpha\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta}\right]\right)^{n} \quad (19)$$

Taking the log of the both sides we get

$$\log L(x_i;\theta) = n\log\alpha + nlo\vartheta + n\log\beta - 2\sum_{i=1}^{n} \log(x_i \square) + \beta\sum_{i=1}^{n}\left(\frac{-\vartheta}{x_i}\right) - (\beta+1)\sum_{i=1}^{n} \log\left(1-\exp\left(-\frac{\vartheta}{x_i}\right)\right) - \alpha\sum_{i=1}^{n} \log\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta}$$

$$\frac{\partial(\log L(x_i;\vartheta))}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} \log\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta} \quad (20)$$

Equating (13)to zero we get

$$\frac{n}{\alpha} - \sum_{i=1}^{n} \log\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)} \square\right)^{\beta} = 0 \quad (21)$$

$$\frac{\partial(\log L(x_i;\vartheta))}{\partial\beta} = \frac{n}{\beta} - \sum_{i=1}^{n}\left(\frac{-\vartheta}{x_i}\right) - \sum_{i=1}^{n} \log\left(1-\exp\left(-\frac{\vartheta}{x_i}\right)\right) - \alpha\sum_{i=1}^{n} \square \left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta} \times \log\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right) \quad (3.37)$$

Equating (3.37) to zero we get

$$\frac{n}{\beta}-\sum_{i=1}^{n}\left(\frac{-\vartheta}{x_i}\right)-\sum_{i=1}^{n} log\left(1-\exp\left(-\frac{\vartheta}{x_i}\right)\right)$$

$$-\alpha\sum_{i=1}^{n}\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta}\times$$

$$log\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)=0 \quad (22)$$

$$\frac{\partial\left(log\ L(x_i;\vartheta)\right)}{\partial\vartheta}$$

$$=\frac{n}{\vartheta}-\beta\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)+(\beta+1)\sum_{i=1}^{n}\left(\frac{\frac{1}{x_i}\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)$$

$$-\alpha\sum_{i=1}^{n}\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta-1}\times$$

$$\frac{\partial}{\partial\vartheta}\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)$$

$$\frac{n}{\vartheta}-\beta\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)+(\beta+1)\sum_{i=1}^{n}\left(\frac{\frac{1}{x_i}\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)$$

$$-\alpha\sum_{i=1}^{n}\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)^{\beta-1}\times$$

$$\frac{\partial}{\partial\vartheta}\left(\frac{\exp\left(-\frac{\vartheta}{x_i}\right)}{1-\exp\left(-\frac{\vartheta}{x_i}\right)}\right)=0 \quad (23)$$

The Maximum Likelihood Estimate of the Exponentiated Generalized Inverse Exponential Distribution (EGIED)

$$L(t_i;\theta)=\prod_{i=1}^{n}\left(f(t_i;\theta)\right)$$

$$\prod_{i=1}^{n}\left\{\frac{\alpha\beta}{t^2}\exp\left(-\frac{\theta}{t.}\right)exp\left(-\frac{\theta}{t}.\right)\right.$$

$$\left.\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha-1}\left(1-\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha}\right)^{\beta-1}\right\}$$

$$=(\theta\alpha\beta)^n\prod\frac{1}{t^2}\exp\left(-\frac{\theta}{t.}\right)exp\left(-\frac{\theta}{t}.\right)$$

$$\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha-1}\prod^{n}\left(1\right.$$

$$\left.-\left[1-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha}\right)^{\beta-1}$$

$$L(t_i;\theta)=nlog\alpha+nlog\beta+nlog\theta-2logt_i$$

$$-\sum_{i=1}\left(\frac{\theta}{t_i}\right)+(\alpha-1)log\left[1-exp\left(-\frac{\theta}{t}.\right)\right]+$$

$$(\beta-1)\sum_{i=1}^{n}log\left(-\left[-exp\left(-\frac{\theta}{t}.\right)\right]^{\alpha}\right) \quad (24)$$

Criteria for Selecting the Best Fitting Models Amongst the Competing models

The goodness of fit tests is used to compare fitted model and determine how well the distributions fit a given data set. To achieve this, the following goodness of fit criteria were considered; -2 Likelihood, Akaike Information Criterion (AIC), Akaike Information Criterion Corrected (AICC) and Bayesian Information Criterion (BIC). According to Palelum (2007) these criteria are computed as follows:

$$AIC = -2log\ L + 2k \quad (25)$$

Where logL is the maximum value of the log-likelihood function under the considered distribution, and k is the number of parameters. Akaike Information Criterion Corrected (AICC) is computed as follows:

$$AICC = AIC + \frac{2k(K+1)}{(n.-k.-1)} \qquad (26)$$

The Bayesian Information Criterion (BIC) is computed as:

$$BIC = -2log\ L + klogn \qquad (27)$$

Data Sets Used for the Study

The efficiency of maintenance or replacement model is examined systematically based on data obtained for performance and technical condition in relation to facilities or parts of facilities. The proposed approach for achieving maximum likelihood estimates of failure parameters would be demonstrated using some failure data.

The data sets used in this study were obtained as a secondary data; they were used to fit into the distributions. Some of the data sets were extracted from journals. All the data sets used are failure data from Engineering. The data sets are given below.

Data Set1: The Time to Failure of 500mw Power Generating Set

The first two failure data set used in this study was given by Xu (2019). The first is the failure time data sets of 500mw power generating set, and the second is the failure data set of a load-haul-dump (LHD) machine. The data are given below.

0.058	0.070	0.090	0.105	0.113	0.121	0.153	0.159
0.224	0.421	0.570	0.596	0.618	0.834	1.019	1.104
1.497	2.027	2.234	2.372	2.433	2.505	2.690	2.877
2.879	3.166	3.455	3.551	4.37 8	4.872	5.085	5.272
5.341		8.952		9.188		11.399	

Data Set 2:The Life-Time of 30 Light Bulbs Rounded to the Nearest Hour.

1122,922,1146,1120,1079,905,1095,977,1138, 1150,977,1137,1088,1139,1055,1082,1053, 1048,1088,996,1102,1028,1130,1002,990,1052, 1116,966,1132,1135.

Data set 3 was given by Looney et al. (2011)

Data Set 3 : The Strength Data of Glass of the Aircraft Window

The fourth data set is the strength data of glass of the aircraft window given by Fuller et al (1994), studied by Shanker (2015). The data set is given below

18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.50, 25.52, 25.80, 26.69, 26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98,37.08, 37.09, 39.58, 44.045, 45.29, 45.381.

Data Set 4: The Number in Million of Revolutions before the Failure of each 23 Deep Groove Ball Bearings in the Life Tests.

The fifth data set is the number in million of revolutions before the failure of each 23 deep groove ball bearings in the life tests. The data set was first studied by Lawlesss (1982), it has also been used by Shanker et al. (2015). The observations are as follows:

2.75,0.13,1.47,0.23,1.81,0.30,0.65,0.10,3.00,1.7 3,1.06,3.00,3.00,2.12,3.00,3.00,3.00,0.02,2.61,2 .93, 0.88, 2.47, 0.28, 1.43, 3.00, 0.23, 3.00, 0.80, 2.45, 2.66

Results

To establish the performance of the selected probability models for the failure time data used in this study, the distributions were fitted to the data sets. The results for the goodness of fit test are shown in the Tables below. The distribution with the smallest AIC, BIC and -2InL is regarded the most flexible and superior distribution for the data sets.

Table 1: Fitted Probability Models of Failure Times for Data Set 1

Fitted Model for Failure Time	Parameter Estimate	-2inL	AIC	BIC	AICC
Weibull	0.816(shape) 2.312(Scale)	137.300	141.381	144.548	141.740
Exponential	0.390(rate)	139.890	141.890	143.473	`
Three-Parameter Weibull Model	0.049(shape) 0.113(Scale) 0.155(Location)	131.292	136.292	139.400	133.040
Generalized WEIBULL Distribution	0.065(shape) 0.002(scale) 0.001(location)	116.500	122.509	127.178	123.250
Weibull Inverse Exponential	0.100(shape) 0.260(scale) 0.003(location)	161.201	167.201	169.200	

Table 2: Fitted Probability Distributions for Failure Times Data 2

Fitted Model for Failure Time	Parameter Estimate	-2inL	AIC	BIC	AICC
Weibull	1.048 (shape) 9.560 (Scale)	334.520	338.521	341.323	337.965
Exponential	rate 0.168	335.260	339.259	348.660	
Three-Parameter Weibull Model	0.004(shape) 0.265(Scale) 0.098 (Location)	333.974	337.975	340.527	848.344
Generalized WEIBULL Distribution	0.004 (shape) 0.002 (scale) 0.001 (location)	159.317	165.317	168.421	928.135
Weibull Inverse Exponential	0.100 (shape) 0.244 (scale) 0.001 (location)	322.368	328.368	332.683	
EGIED	0.00 1 (shape) 0.003 (scale) 0.046 (location)	203.052	209.053	302.143	

Table 3 : Fitted Probability Distributions for Failure Times Data 3

Fitted Model For Failure Time	Parameter Estimate	-2inL	AIC	BIC	AICC
Weibull	4.635 (shape) 33.673 (Scale)	210.978	214.978	217.846	215.406
Exponential	0.033 (rate)	274.529	276.529	277.963	279.044
Three-Parameter Weibull Model	0.012 (shape) 0.234 (Scale) 0.057 (Location)	212.120	214.120	216.008	215.348
Generalized WEIBULL Distribution	0.065 (shape) 0.002 (scale) 0.001 (location)	102.591	102.592	107.095	103.481
Weibull Inverse Exponential Distribution	1.000 (shape) 0.242 (scale) 0.010 (location)	178.282	184.283	193.050	

Table 4: Fitted Probability Distributions for Failure Times Data 4

Fitted Model For Failure Time	Parameter Estimate	-2inL	AIC	BIC	AICC
Weibull	1.265 (shape) 1.880 (Scale)	92.316	96.318	99.119	97.671
Exponential	rate 0.565	94.270	96.270	99.119	
Three-Parameter Weibull Model	0.713(shape) 0.145 (Scale) 0.145 (Location)	79.816	85.817	89.516	
Generalized WEIBULL Distribution	0.065 (shape) 0.002 (scale) 0.001 (location)	99.817	95.169	97.188	
Weibull Inverse Exponential Distribution(WIE)	0.010 (shape) 0.269 (scale) 00.001(location)	240.074	246.074	377.890	388.900

Discussion/Conclusion

Table 1 reveals the result of the parameter estimate and goodness of fit for the failure time of 500mw power generating set (data set 1). Three-Parameter Weibull and Generalized Weibull distribution gave adequate fit to the data compared with the Classical Weibull and exponential distributions. The Generalized Weibull distribution gave the best fit to the data set.This implies that the Generalized Weibull distribution is an improvement over the classical Weibull for failure data set. The Weibull, Exponential, Three-Parameter Weibull, and the Generalized Weibull distributions were fitted to the third failure time data (data set 2), the parameter estimate and goodness of fit results are presented in Table 2.The result in Table 2 revealed that, the Three Parameter Weibull and the exponentiated inverse exponential distributions gave good fit to the data with the Generalized Weibull distribution providing the best fit amongst all distributions based on all the criteria. Table 3 reveals the result of the parameter estimate and goodness of fit for the data. The Generalized Weibull distribution gave the best fit to the dada. This implies that the Generalized Weibull distribution is an improvement over the classical Weibull for failure data set.

Table 4 shows the result of the goodness of fit of the Weibull, Exponential, Three-Parameter Weibull, the Generalized Weibull distributions for the forth failure data.The result revealed that, the Generalized Weibull Distribution (GWD) and Three Parameter Weibull gave good fit to the data with the Three Parameter Weibull providing the best fit amongst all distributions based on all the criteria. This implies that the Three parameters Weibull distribution is an improvement over the classical Weibull distribution for this failure data set.

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