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APPLICATION OF MAINTENANCE AND REPLACEMENT MODELS TO ENHANCE EFFICIENCY OF EQUIPMENT.

OBELE, REALMAN EVANS[1]
&
GEORGE, ISOBEYE[2]
Department of Mathematics[1&2],
Ignatius Ajuru University of Education,
Port Harcourt, Nigeria.

Abstract
In this study, the economic years of two equipment used in the production process of a certain company were determined using replacement and maintenance models. Two real life data sets on annual cost of maintenance and duration of the equipment under study were used. The replacement model with constant money value and an increasing running cost was used. The result of the study reveals that the economic life of the equipment is seven years. Thus, the optimum replacement policy was to replace the equipment after seven years.

Keywords: Application, maintenance, replacement, models, efficiency, equipment.

Introduction

Factories and firms which provide goods and services make use of several equipment and machines including humans, which are needed in the productions process. Such equipment, machineries and other facilities deteriorates or becomes ineffective or less effective with the passage of time, giving rise to the problem of maintenance or replacement as remedial measures to bring the item or equipment to its original level or status (Anders, 1990). Determining when to replace assets is one of the significant problems been faced by asset management staff (Lebow&Vinberg, 1998).

This seems to be a difficult issue, which calls for the formulation of several mathematical models with quite a lot of parameters such as operating costs, maintenance cost and information from technical systems and amongst others (Anders et al., 2001).
Organizations such as paper, energy and pulp industries etc., maintaining equipment could be highly capital intensive. The revitalization of some technological equipment could be very expensive as well. On the other hand, the life span of equipment varies to a great extent, since the equipment consist of varying residual life span. Some systems operate for only a few years while others can have a longer life span. If a company must remain in business in the face of competition, it has to decide whether or not to replace the aged equipment or to maintain it by taking the cost of maintenance and operation into account and determining the accurate economic years of the equipment (Baron & Pate-cornell, 1999).

By replacing ineffective equipment or items with new ones at frequent interval, maintenance and other overhead cost could be reduced or increase, such replacement might also increase the capital cost for new ones. The replacement model is concerned with the situations that arise when some items such as machines, electric light bulbs, computer, etc. need replacement due to their decreased efficiency, failure or break-down. Such decreased efficiency or complete breakdown may either be gradual or all of a sudden. The replacement problem arises mainly because of the following reasons: The aged equipment works faultily or involves expensive maintenance cost; old equipment has failed owing to accident or otherwise and does not work at all (Balzer et al., 2001).A better or more efficient design of machine or equipment has become available in the market. In this study, the replacement occasioned by excessive running cost of an item is considered.

According to Suzan (2012) repairable systems modeling encompasses the application of operations research techniques such as probability modeling, optimization, and simulation to solve problems associated with the replacement and maintenance of equipment. Replaceable system models are usually utilized in the estimation of the performance of one or more repairable systems and the design of appropriate maintenance policies for one or more equipment. The literature on the use of mathematical modeling for the purpose of analyzing and optimizing the performance of repairable systems is extensive.

An inaccurate estimate of the left-over life span or economic years of an equipment might result in early replacement or renewal which is associated wastage of resources and high capital costs (Enogwe, 2018).

On the other hand, if replacement or maintenance is delayed, a breakdown might take place which could lead to a serious damage to technical equipment and a loss of income due to outages. For instance, in the energy industry a breakdown might as well lead to problems with the local community. Therefore, early replacement seems to be a better option (Campbell, 2001) in order to obviate the occurrence of major breakdowns (Billington et al., 2000).

From the forgoing, replacement and maintenance of equipment is an important part of the whole operating costs in a lot of companies, industries and other sectors (Murthy, 2002).

Pintelton (1992) noted that an effective maintenance and replacement management involves a multi disciplinary approach where replacement is seen from business perspective. There is a modification in the approach and technique adopted in maintenance in recent times. In the recent approach, replacements or renewals are the result of a rigorous process, which takes cognizance of all the pertinent circumstances as well as the technical aspects, for instance, the cost of procuring brand-new equipment, the running cost of the existing equipment, environmental factors and so on.

The decision whether to renew or not is then taken, based on a balancing of all these facts. As part of a technical system frequently has to be kept in service, maintenance activities are frequently carried out to keep the system working.

A lot of maintenance models and theories exist in literature, on the other hand, these models and methods provides limited decision when it comes to the appropriate replacement timing and financial consequences of neglecting maintenance. Most of these challenges could be addressed by the operations research approach. Existing methodologies have their main focus, some are geared towards a specific equipment and are unable to give a generalized information about replacement time of other equipment (Andersen et al., 1998; Gammelgård, 2003).

Maintenance or replacement is expected to be carried out in such a way that cost is minimized and profit is maximized. Because of the long operative lifetime of some equipment, determining slow deterioration of the technical systems might be difficult. Maintenance and replacement models focus on these shortfalls (Leite da Silva et al., 2001). If only short-term results are considered and maintenance is cut down, there might be a risk that, slow deterioration might occur, resulting in high need for resources in the future during a short time span.

The purpose of the replacement model is to monitor and forecast breakdown and to detect early deterioration. The application of mathematical models for the optimization of the performance of replacement systems is under researched in mathematical literature. In this research, optimizing a repairable system is studied. The failure rate of a system has been broadly classified as follows:

Sudden failure: These types of failures occur in equipment after some period of desired service rather than weakening while in service. The time of desired service is not constant but follows some frequency distribution. Unexpected breakdown may be progressive, retrogressive or random in nature.

Progressive failure: Failure is said to be progressive if the likelihood of the failure of an equipment increases with the increase in the usage or life of the equipment, then such a failure is called progressive failure. For example, light bulbs and tubes fail progressively.

Retrogressive failure: In a retrogressive failure, the probability of failure at the commencement of the usage of an equipment is higher, and reduces as the age of the equipment increases.

Random failure: In a random failure, the probability of failure is not connected with the length of time the equipment as been used. That is, failure rate is independent of age of equipment. The replacement situations may be placed into the following two main categories;

(i) Replacement of equipment that depreciates with time examples are planes, buses, equipment, machine and other tools.
(ii) Individual or group replacement of items that falls short entirely, for instance light bulbs, tubes.

From the forgoing, it could be deduced that the issue of replacement takes place when a unit performing a task, such as machines, men, equipment, and so on, become less effective or useless due to either sudden or gradual deterioration in their efficiency, failure or complete breakdown. For instance, a machine suddenly develops a fault, an electric lamp fails etc. These are some examples of circumstances that require an optimum replacement strategy to restore efficiency. If a machine performs with decline in efficiency, subsequently it might require an additional cost to increase the operating efficiency, such cost include cost of repair or maintenance cost (Nwabueze, 2003). Thus, in this paper, the economic life of machines is predicted using operations research maintenance/replacement model.

Statement of the Problems:
Wrong estimation of the remaining life-span or economic years of an equipment might result to early replacement or maintenance, which may end up in wastage of scare resources in organization. In the same vein, if maintenance or replacement intervals are miss-calculated, it can result to breakdown which could lead to a serious damage to the equipment and loss of income due to equipment downtime. Therefore maintenance or replacement should be carry out in a manner that cost is minimized and profit maximized. Maintenance and replacement Models can be used to overcome these shortfalls in the industry.

Aim and objectives of the study:
The aim of this paper is to determine the optimum time to replace a repairable system using operations research models. The following are the objectives of this study:

i. Apply the replacement model for equipment that running cost increase with time when money value is constant.
ii. Determine the optimum replacement policy for the equipment in (I) above.

Methodology
The following notations are used in the study:

AT(n) = The average of the total annual cost

S = Resale or Scrap value of an equipment

CC = Capital cost of an equipment

N = Length of time the equipment will be in use

F (t) = Cost function of maintenance at a time t

In the replacement model, two possibilities are considered for the time:

i. When the timet is continuous random variable.
ii. When time t is discrete random variable.

When the time 't' is continuous random variable

According to Barlow (1975) the annual cost of the item at any time t is F (t) + CC – S

If the time is considered a continuous random variable, the cost of maintenance in n years is given by the following equations (Barlow, 1975):

$$\int_0^n F(t) \quad (1)$$

The total cost involved in n years is given as

$$T(n) = \boldsymbol{CC} + \int_0^n F(t) - \boldsymbol{S} \quad (2)$$

The average cost is given by

$$AT(n) = \frac{1}{n}\left(\boldsymbol{CC} + \int_0^n F(t) - \boldsymbol{S}\right) \quad (3)$$

$$= \frac{\boldsymbol{CC} - \boldsymbol{S}}{n} + \frac{1}{n}\int_0^n F(t) \quad (4)$$

It is desire to determine the optional value of n which if the value of n for which *AT (n)* is minimum

The minimum cost = $\frac{\partial}{\partial n} AT(n) = 0$

$$-\frac{\boldsymbol{CC} - \boldsymbol{S}}{n^2} + \frac{1}{n} F(n) - \frac{1}{n^2}\int_0^n F(t) = 0 \quad (5)$$

$$F(n) = \frac{\boldsymbol{CC} - \boldsymbol{S}}{n^2} + \frac{1}{n}\int_0^n F(t) \quad = AT(n) \quad (6)$$

$\frac{\partial^2}{\partial n^2} AT(n) > 0$ if and only if $AT(n) = F(n), \quad n \neq 0$

The following are deduced from the above equations.

This reveals that the cost of maintenance at time n = average total cost in time n.

Thus, when the maintenance cost becomes equal to the average annual cost, the decision should be to replace the equipment.

When time ‘t’ is a discrete variable

If the time ‘t’ is taken to be discrete random variable, the cost equation could be expressed as follows (Barlow, 1975):

$$AT(n) = \frac{CC - S}{n} + \frac{1}{n}\sum_{t=1}^{n} F(t) = F(n) \quad (7)$$

Equation (7) represents the average cost over n period

$CC - S$ is assumed to be monotonically decreasing while F (t) is monotonically increasing.

This implies that there exists an n value for which $AT(n)$ is minimum.

It also follows that:

$$AT(n-1) > AT(n) < AT(n+1)$$

$AT(n)$ is minimum if

$$AT(n+1) > AT(n) \text{ and } AT(n-1) > AT(n) \quad (8)$$

Which also implies that $AT(n)$ is minimum if

$$AT(n+1) - AT(n) \geq 0 \text{ and } AT(n-1) - AT(n) \leq 0 \quad (9)$$

$$AT(n+1) - AT(n) = \frac{CC - S}{n+1} + \frac{1}{n+1}\sum_{t=1}^{n} F(t) + \frac{1}{n+1} F(n+1) \quad AT(n) \quad (10)$$

$$= \frac{1}{n+1} F(n+1) + \frac{n}{n+1} AT(n) - AT(n) \geq 0$$

Since $AT(n+1) - AT(n) > 0$

$$\Rightarrow AT(n) \leq F(n+1) \text{ or } F(n+1) \geq AT(n) \quad (11)$$

Similarly,

$AT(n-1) - AT(n) \geq 0$ Implies that

$$F(n) \leq A(n-1) \quad (12)$$

The following optimal policies are made from equations (11) and (12)

To minimize cost, replace the items at the end of the year n if the cost of maintenance in the (n+1)th year is above the average total cost in the nth year and the maintenance cost in the nth is smaller than preceding year's average total cost.

Results

In this section, an application of the replacement model is carried out. The data used for this study is a real-life data set reported by Bayzid (2014). The problem is to determine the optimal replacement policy for the equipment.

Table 1
Data on Maintenance and Purchase Price of Equipment Wheel Loaders (4cy)

Manufacturer	**Year**	**Hour Meter Reading**	**Procurement Price($)**	**Maintenance Cost ($)**
Komatsu	2001	11093.00	127,500.0	9.54
Komatsu	2002	12081.00	127,500.0	12.75
Komatsu	2003	12694.00	127,500.0	47.14
Komatsu	2004	13840.00	127,500.0	33.60
Komatsu	2005	14958.00	127,500.0	37.65
Komatsu	2006	22374.00	127,500.0	37.82
Komatsu	2007	-	127,500.0	37.89
Komatsu	2008	-	127,500.0	38.21
CAT	2011	1507.00	304,631.140	8.04
CAT	2012	1713.00	304,631.140	13.28
CAT	2013	2168.00	304,631.140	12.19
CAT	2014	2744.00	304,631.140	9.55
CAT	2015	3644.00	304,631.140	17.55
CAT	2016	3897.00	304,631.140	16.71

From Table 1, the equipment manufactured by komatsu, we assume a resale value of $ 15300, CC= $127,500.0. For the CAT, the scrap value is $45694.671

Table 2
Depreciation and average Running Cost for the Komatsu Equipment.

Year	Meter Reading	Procurement Price($)	Maintenance Cost MC($)	Cumulative MC	T(n)=CC+ $\sum F(n) - s$	AT(n)
01	11093.00	127,500.0	1,284.63	1,284.63	113484.63	113484.63
02	12081.00	127,500.0	7,608.88	8893.51	121093.51	60546.755
03	12694.00	127,500.0	9,000.21	17893.72	130093.72	43364.5733
04	13840.00	127,500.0	10,073.19	27966.91	140166.91	35041.7275
05	14958.00	127,500.0	15,951.99	43918.90	156118.9	31223.7800
06	22374.00	127,500.0	19,132.86	63051.76	175251.76	29208.6267
07	-	127,500.0	21,222.23	84273.99	196473.99	**28067.7129**
08	-	127,500.0	32,231.22	116505.21	228705.21	28588.153

Table 2 shows the average running cost of the Kumatsu equipment. Firstly, the average running cost per annum was computed during the life of the equipment using equation (7). The Total cost in first year = Purchase – resale (scrap) value + maintenance cost in the year. Hence, the average cost in the first year is $113484.63. The total cost up to two years = running cost up to two year + Purchase – resale (scrap) value. Consequently, average cost in first two years = $60546.755. Similarly, the average running cost per annum in the first three years is $43364.5733 and so on.

Furthermore, it is observed that the average running cost in the 7th year is smaller than the average cost in the 6th and 8thyears, that is, the average maintenance cost increase after the seventh year. Thus, the inequality in (11) and (12) are satisfied. Therefore, the optimal decision is to replace the equipment at the end of the 7th year. On the other hand, last column of Table 2 shows that the average cost starts increasing in the 7th year. This implies that the equipment should be replaced before the 8th year begins. i.e., at the end of the 7th year.

Table 3

Depreciation and average Running Cost for the CAT Equipment.

Product	year	Maintenance Cost MC ($)	Procurement Price($)	Cumulative MC	T(n)=CC+ $\sum F(n) - s$	AT(n)
CAT	01	1507.0	304,631.14	1507.0	260443.469	260443.469
CAT	02	1713.0	304,631.14	3220	262156.469	131078.236
CAT	03	2168.0	304,631.14	5388	264324.469	88108.1563
CAT	04	2744.0	304,631.14	8132	267068.469	66767.11725
CAT	05	19644.0	304,631.14	27776	286712.469	57342.494
CAT	06	2523.0	304,631.14	30299	289235.469	48205.9115
CAT	07	37154	304,631.14	67453	326389.469	**46627.067**
CAT	08	49232	304,631.14	116685	375621.469	46952.684
CAT	2019	51532	304,631.14	168217	427153.469	47461.4966

Table 3 shows the average running cost of the CAT equipment. Firstly, the average running cost per annum was computed during the life of the equipment using equation (7). The Total cost in first year = Purchase – resale (scrap) value + maintenance cost in the year.

Hence, the average cost in the first year is \$260443.469. The total cost up to two years = running cost up to two year + Purchase – resale (scrap) value. Consequently, average cost in first two years = \$131078.236. Similarly, the average running cost per annum in the first three years is \$88108.1563etc.

Furthermore, it is observed that the average running cost in the 7th year is smaller than the average cost in the 6th and 8^{th}years, that is, the average maintenance cost increase after the seventh year. Thus, the optimal decision is to replace the equipment at the end of the 7th year. Alternatively, last column of Table 3 shows that the average cost starts increasing in the 7th year. This implies that the equipment should be replaced before the 8th year begins. That is at the end of the 7th year.

Discussion of Findings.

From Tables 2 and 3,it is obvious that the cumulative maintenance cost increases alongside the life of the equipment, while the average running cost or average maintenance cost keeps on decreasing with the life of the equipment. From the beginning, the total cost keeps on decreasing until a particular point in time when it starts increasing. The point where the total cost is minimum is called the economic life of the machine. In order to minimize cost and maximize profit we seek the economic life of the equipment. The results in Table 2, reveals that the average running cost of the Komastu equipment decreases with the least total running cost at the 7th year after which the average total running cost increases. Satisfying the inequality in (11) and (12).

Thus, we conclude that the economic life of the equipment understudy is seven years. An optimal maintenance policy would be to replace the equipment after the seventh year.

Similarly, Table 3 reveals that the economic life of the Cat Machine is seven years. This suggests that replacing the CAT equipment after the seventh year will save cost. Furthermore, Tables 3 and 4 reveals that the cumulative running cost of the equipment increase over the years, which is in line with the findings of Enogweetal. (2018).

In order to determine the length of time to carry out maintenance on an equipment, it is necessary to carry out some reliability test on the equipment. A lot of approaches exist for such analysis. According to Nwabueze (2003)two basic replacement models are available in literature, specifically we have; replacement models for equipment that gradually fails with passage of time and the model for equipment that fails unexpectedly. The former was considered in this study. Considerable efforts have been made by researchers in addressing the problem of replacement of items that fail gradually with the passage of time. In this study the best time to replace a machine has been determine using operations research approach using the discrete time replacement model with a constant money value. It is deduced from the findings of this study that the optimal replacement time for the equipment so far considered is seven years.

Conclusion

An inaccurate replacement/renewal policy could be accompanied with high capital costs. On the other hand, delay in replacement might lead to breakdown which could lead to some damages and a loss of income due to outages. As a result of these complexities, optimal policies are needed.

The replacement model is concerned with situations that take place when machines, equipment, computer, and so on require replacement as a result of their reduced efficiency, high maintenance cost, and breakdown or failure. This work demonstrates the use of the operations research maintenance/ replacement model in the determination of optimum replacement time.

In This work the application of the replacement model for equipment with increasing running cost is demonstrated. The aim was to recommend the optimal replacement policy for the equipment studied in this research. Using the models, the economic life of the machines was found to be seven years.

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